

4. Positron Annihilation-in-Flight

The absolute intensity of 511-keV photons per 100 disintegrations ($\gamma^\pm(\%)$) from positrons annihilating at thermal energies in an absorber is:

$$\gamma^\pm(\%) = 2[\beta^+(\%) - \beta_f^+(\%)] , \quad (1)$$

where $\beta^+(\%)$ and $\beta_f^+(\%)$ are the emitted and annihilated-in-flight absolute positron intensities, respectively.

There is a significant probability for annihilation-in-flight to result in *two quantum annihilation* (TQA) or *one quantum annihilation* (OQA) with a continuous photon energy distribution. The maximum photon energy is E_0+1 , where E_0 is the maximum positron kinetic energy (endpoint) in units of the electron rest mass $m_e c^2$. The OQA probability for annihilation-in-flight of a positron of energy E by collision with an atomic electron is given by Bethe¹ as

$${}^{OQA}\Phi(E,Z) = \frac{2\pi\alpha^4 Z^5 r_0^2 \left[E^2 + \frac{2}{3}E + \frac{4}{3} - (E+2) \ln[E + (E^2 - 1)^{1/2}] (E^2 - 1)^{-1/2} \right]}{(E + 1)^2 (E^2 - 1)^{1/2}} , \quad (2)$$

and the TQA probability, as

$${}^{TQA}\Phi(E) = \frac{\pi r_0^2 \left[(E^2 + 4E + 1) \ln[E + (E^2 - 1)^{1/2}] - (E + 3)(E^2 - 1)^{1/2} \right]}{(E^2 - 1)(E + 1)} , \quad (3)$$

where $r_0 = 2.82 \times 10^{-13} \text{ cm}$ is the classical electron radius, $\alpha \approx 1/137$ is the fine structure constant, and Z is the atomic number of the absorber. Positron energies are given in units of the electron rest mass ($m_e c^2$).

The OQA probability given in equation (2) is generally small, except for high- Z absorbing materials where it is $\approx 16\%$ that of the TQA probability. Equation (2) includes collisions with electrons from the K atomic shell only. The TQA probability given in equation (3) includes collisions with electrons from all atomic shells. The total probability for annihilation-in-flight by positrons of energy E is given by²

$$P(E,Z) = \frac{N\rho}{A} \int_0^E [Z {}^{TQA}\Phi(E) + 2 {}^{OQA}\Phi(E,Z)] (-dE/dx)^{-1} dE , \quad (4)$$

where N is Avogadro's number, A , ρ and dE/dx , are the atomic weight, the density, and the stopping power of the absorber, respectively.

Figure 7 shows the total probability for annihilation-in-flight of fully absorbed positrons in Be, Al, Cu, Ag, Pb and U, calculated with equation (4). The integration was done numerically and the probability corrected for the fact that a positron that has already annihilated can not reappear at a lower energy. The stopping power, (dE/dx) , was calculated for both collision- and bremsstrahlung-energy losses; for collisions, as described by Nelms,³ with mean excitation energies and density-effect corrections of Sternheimer⁴; for bremsstrahlung, as described by Koch and Motz.⁵

¹H.A. Bethe, *Proc. Roy. Soc. Lond.* **A150**, 129 (1935).

²G. Azuelos and J.E. Kitching, *At. Data and Nucl. Data Tables* **17**, 103 (1976).

³A.T. Nelms, *Energy Loss and Range of Electrons and Positrons*, NBS Circular 577 (1956).

⁴R.M. Sternheimer, *Phys. Rev.* **103**, 511 (1956).

⁵H.W. Koch and J.W. Motz, *Rev. Mod. Phys.* **31**, 920 (1959).

Figure 7. Probability for annihilation-in-flight of fully absorbed positrons in various media

